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CCD Astrometry for Amateurs

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Abstract

Because reasonably priced CCD detectors are available, amateurs can now make astrometric observations of very high quality. In particular, there is a continual need for observations of asteroids and comets in order to improve their orbits and to better understand their dynamics. A recent paper by Meyer and Raab (1995) describes in very simplified terms how these observations and reductions should be made. However, they overlooked various considerations that are important for good astrometry, namely, not all combinations of telescope focal lengths and pixel sizes are suitable, nor are all choices for a passband. Moreover, astrometry using only three reference stars, which they claim is acceptable, can cause large positional errors. This paper describes the subject in more detail, including a discussion as to how CCD coordinates can be reduced to equatorial coordinates, which amateurs might find interesting. Currently, amateurs are determining the positions of solar system objects accurate to ±0.6 arcsec with the Guide Star Catalog. In a few more years, the Tycho star catalog will become available, and accuracies of ±0.1 arcsec should become possible.

1 Introduction

Amateurs have a long history of contributing to astronomy, mainly in the areas of stellar photometry and occultation studies. In the past, astrometry has not been an active area for amateur research, since until recently, photographic plates and their subsequent measurement were needed before star positions could be accurately determined. Photographic plates are expensive and their processing requires machines capable of measuring star positions to several microns or better. This situation has drastically changed. As discussed by Holmes (1995), high quality charge-coupled-devices (CCDs) are now available to amateurs that are reasonably priced. These devices have formats as large as 1000 x 1000, pixel sizes of 10-20 microns, readout noises under 15 electrons, and low dark currents. Moreover, thermoelectric cooling to about -30°C and a wide variety of PC-interfaces and software support packages are also available to amateurs. Among professional astronomers, CCDs have largely replaced other types of detectors because of their linearity, high quantum efficiency, and digital output (see Kristian and Blouke 1982,

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14. ABSTRACT
Because reasonably priced CCD detectors are available, amateurs can now make astrometric observations of very high quality. In particular, there is a continual need for observations of asteroids and comets in order to improve their orbits and to better understand their dynamics. A recent paper by Meyer and Raab(1995) describes in very simplified terms how these observations and reductions should be made. However, they overlooked various considerations that are important for good astrometry, namely, not all combinations of telescope focal lengths and poxel sizes are suitable, nor ar eall choices for a passband. Moreover, astrometry using only three refernece stars, which they claim is acceptable, can cause large positional errors. This paper describes the subject in more detail, including a discussion as to how CCD coordiantes can be reduced to equatorial coordinates, which amateurs might find interesting. Currently, amateurs are determining the positions of solar system objects accurate to=0/6 arcsec with the Guide Star Catalog. In a few more years, the Tycho star catalog will become available, and accuracles of=0.1 arcsec should become possible.

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Mackay 1986, and Janesick and Blouke 1987, for more details). Since CCDs have eliminated the need for photographic plates and their measurement in most applications in astrometry, quality astrometry can now be done outside of the professional community.

Astrometry is often used to determine the proper motions and annual parallaxes of stars. Unfortunately, these programs require large telescopes, and many years of observation are needed before meaningful results can be derived. Consequently, participation in these areas would be difficult for amateurs. On the other hand, asteroids and comets have large relative motions, and there is a continual need to improve their orbits. Astrometric observations of asteroids are needed to understand their dynamics, and observations of comets are needed to understand how non-gravitational forces influence their orbits. The Minor Planet Center at the Smithsonian Astronomical Observatory (Marsden 1988) needs astrometric data of these objects, and amateurs are now contributing positions determined from CCD data. These positions are comparable in accuracy with those determined by professionals using the currently available reference star catalogs. Meyer and Raab (1995) discuss amateur contributions in these areas.

Because of its high density, the Guide Star Catalog (GSC) is the catalog used most often to reduce astrometric observations of asteroids and comets to positions. As discussed by Russell *et al* (1990), the GSC is not particularly accurate ($\sigma \sim \pm 0.6$ arcsec), but on the other hand, includes over 19 million stars with magnitudes ranging from V ~ 9 to 15 mag. The GSC is known to have large systematic errors, and Lopez and Yazudin (1995) give numerical corrections for many of these errors. In a few years, the Hipparcos and Tycho catalogs of star positions will become available (Lindegren *et al* 1992 and Høg 1992), and these will supersede all preceding optical star catalogs in terms of accuracy. The Hipparcos catalog includes about 118,000 stars with a nominal accuracy of ±2 milliarcsecond (mas) in each coordinate, while the Tycho catalog includes about 1 million stars accurate to ±30 mas. Efforts are being made to greatly densify these catalogs in that they will be used to calibrate deep photographic surveys (Morgan *et al* 1992 and Monet *et al* 1994b), and as a result, catalogs including 10^9 or more stars with positions accurate to ±0.2 arcsec will eventually become available. Future observing programs are planned that will use mosaics of CCDs to scan the sky and produce catalogs of star positions to a limit of V ~ 21 mag (for example, see Gunn and Knapp 1993). These future catalogs will greatly increase the accuracy of CCD solar system observations.

Recently, Meyer and Raab (1995) gave a discussion of CCD amateur astrometry. Although the paper is well written and includes many important discussions, it gives nonetheless a simplified approach to CCD astrometry and fails to mention, or adequately describe, a number of important considerations. For example, they do not adequately point out that not all telescope combinations of focal length and CCD pixel size are suited for good astrometry, and the telescope passband should be chosen in the visual or red in order to reduce greatly errors caused by atmospheric refraction. Moreover, they maintain that good astrometry is possible with a minimum of three reference stars. In reality, the number should be larger, because of individual errors in the positions of reference stars. This paper will discuss the basic requirements needed for accurate astrometry in much more detail. Moreover, many amateurs might want to understand the methods used for processing CCD digital data into equatorial star positions. In fact, amateurs can use the procedures described in this paper to develop their own astrometric reduction software. The Meyer and Raab paper only describes a commercially available software package for making astrometric reductions.

2 Requirements for Astrometry

Not all telescopes are suitable for accurate astrometry. In particular, the pixel size needs to be small enough to adequately resolve stellar images, while the CCD field size needs to be large enough to include many reference stars taken from available star catalogs. A lack of reference stars can distort solutions for star positions, and poorly resolved images can cause large errors when stars are centred. Long focal length telescopes can resolve stellar images well, but often have small fields. On the other hand, small focal length telescopes have wide fields but poor resolutions. This section will discuss which combinations of telescope focal lengths pixel sizes, and CCD formats are needed for good astrometry.

Tables I and II have been prepared to show which telescopes are suitable for astrometry. In this paper, good astrometry is considered possible with a given telescope if it has a pixel size p smaller than the mean seeing disk ø at the observing site (p < ø), and the CCD subtends an area in the sky large enough to include at least 10 reference stars taken from either the GSC or Tycho catalogs. As indicated from the simulations discussed in Stone (1989), centring errors become significant with poorer resolved stellar images. As will be subsequently discussed, three unknowns are needed to solve for the transformations between CCD and standard coordinates. In theory, only three reference stars are necessary, but in reality, more stars are needed to reduce statistical errors. For example, if three reference stars are used and one has a poor position determined for it, then the resulting position for the target object could be grossly in error. A minimum of 10 reference stars can greatly reduce these errors.

Table I. Pixel size in arcseconds.

FL	Pixel Size			
(mm)	10μ	15μ	20μ	25μ
500	4.13	6.19	8.25	10.31
1000	2.06†	3.09	4.13	5.16
1500	1.38†‡	2.06†	2.75	3.44
2000	1.03†‡	1.55†	2.06†	2.58
2500	0.83†‡	1.24†‡	1.65†	2.06†

† suitable for 2.5 arcsec seeing
‡ suitable for 1.5 arcsec seeing

Table I gives the pixel size in arcsec for various assumed pixel sizes in microns and telescope focal lengths (FL). Smaller pixel sizes are not currently available for amateurs, but may become available in the near future. Entries in the table marked with (†) are stellar images with ø = 2.5 arcsec seeing that are well enough resolved for good image centring. Those identified with (‡) correspond to a seeing of ø = 1.5 arcsec. As Table I clearly shows, telescopes with short focal lengths (FL ≤ 500 mm) are not well suited for astrometry. Their resolutions can be improved by defocusing the telescope; however, great care should be taken so that the defocused images retain their Gaussian profiles.

Table II. CCD field size in degree^2.

FL (mm)	Pixel Size 10μ	15μ	20μ	25μ	Pixel Size 10μ	15μ	20μ	25μ
	256 x 256 Array				512 x 512 Array			
500	0.086	0.194	0.344	0.538	0.344	0.775	1.377	2.151
1000	0.022	0.048	0.086	0.134	0.086†	0.194	0.344	0.538
1500	0.010	0.022	0.038	0.060	0.038†	0.086†	0.153	0.239
2000	0.005	0.012	0.022	0.034	0.022	0.048†	0.086†	0.134
2500	0.003	0.008	0.014	0.022	0.014	0.031	0.055†	0.086†
	1024 x 1024 Array				2048 x 2048 Array			
500	1.377	3.098	5.508	8.606	5.508	12.392	22.031	34.423
1000	0.344†	0.775	1.377	2.151	1.377†‡	3.098	5.508	8.606
1500	0.153†	0.344†	0.612	0.956	0.612†‡	1.377†‡	2.448	3.825
2000	0.086†	0.194†	0.344†	0.538	0.344†	0.775†‡	1.377†‡	2.151
2500	0.055†	0.124†	0.220†	0.344†	0.220†	0.496†	0.881†‡	1.377†‡

† suitable for GSC astrometry
‡ suitable for Tycho astrometry

Table II gives the field size for various assumed telescope focal lengths and CCD array sizes. The seeing is assumed to be 2.5 arcsec in the table. The large 2048^2 format CCDs are not currently available in the price range of amateurs, but might become so in the near future. Telescopes are identified in the table meeting the previous criterion for resolution ($p < ø$) and having fields large enough to include 10 GSC (†) or Tycho (‡) catalog stars. As seen, the requirements are less severe when using the GSC catalog, which is not surprising, since it contains many more stars. As previously discussed, very high density star catalogs with accuracies of $\sigma \sim \pm 0.2$ arcsec or better are being prepared, which will enable all but the shortest focal length amateur telescopes to produce accurate astrometry. If the entry for a given telescope equipped with a certain CCD is tagged in Table II, then it can be used for accurate astrometry, otherwise not. With the coming availability of CCD with smaller pixels and larger array sizes, many more telescopes will qualify.

Because of the selective nature of atmospheric refraction, astrometry is more accurate at longer wavelengths, and accordingly, the passband for the telescope should be filtered for visual or longer wavelength light. The sensitivity of the telescope will not be greatly reduced, since CCDs have their greatest response in these spectral regions. If photometry is intended along with the astrometry, then standard passbands (such as those shown in Figure 1) should be chosen.

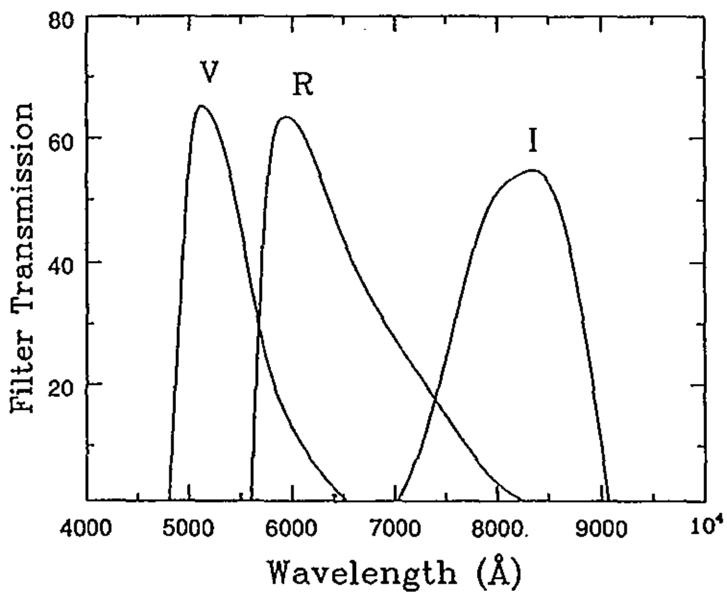

Figure 1. Response curves are shown for commonly used photometric passbands suitable for astrometry.

3 Processing of Digital Data

The reference stars on a particular CCD frame can be easily identified from their known catalog positions, while the target object (eg. asteroid or comet) can often be identified as an image that is slightly elongated. However, slow moving objects can appear as round stellar images, making them difficult to identify on a single CCD frame. By taking multiple CCD frames spaced several minutes apart, a slow moving target object can be easily identified from its relative motion. For the best astrometry, each target object should be surrounded by reference stars on the CCD frame.

After the target and reference stars are identified, each image should be centred accurately. As discussed by Stone (1989), there are many methods for centring star images. Functional fit centring algorithms, such as those discussed by Chui (1977) and van Altena and Auer (1975), are very accurate, but require well resolved images and complicated centring techniques. Non-parametric centring routines are much simpler and give comparable accuracy for well exposed images. For example, Stone (1989) discusses a modified moment routine which is very straightforward, highly convergent, and accurate. The routine will be briefly described. First, a subraster including each object and surrounding sky counts is extracted. As discussed in Stone, a box dimensioned to four times the seeing disk in each coordinate, centred on the object, is adequate. Within each subraster, there is an array of counts $I(x, y)$, corresponding to pixel locations (x, y). For the modified moment, this array is replaced with the array $I'(x, y)$, such that

$$I'(x,y) = I(x,y) - b \quad \text{for} \quad I(x,y) \geq T \quad \text{and} \tag{1}$$

$$I'(x,y) = 0 \quad \text{for} \quad I(x,Y) < T \quad , \tag{2}$$

where T is an adopted threshold. The threshold can be chosen to be $T = b + 3\sqrt{b}$, where b is the local sky background level which can be determined by averaging count levels in the extreme corners of the subraster. Marginal distributions are then formed by summing the counts along columns and rows, and the resulting marginal distributions are given below.

$$M(x) = \sum_{y_1}^{y_2} I'(x,y) \tag{3}$$

$$M(y) = \sum_{x_1}^{x_2} I'(x,y) \tag{4}$$

The marginals reduce the two-dimensional data to more manageable one-dimensional vectors. The centre for each object (x_c, y_c) can be then easily determined with

$$x_c = \frac{\sum_{x_1}^{x_2} x\, M(x)}{\sum_{x_1}^{x_2} M(x)} \quad , \quad y_c = \frac{\sum_{y_1}^{y_2} y\, M(y)}{\sum_{y_1}^{y_2} M(y)} \quad . \tag{5, 6}$$

Well exposed star images can be centred to ±10 mas with this technique. The actual error will be larger because of seeing effects, telescope guiding errors, and errors in the chosen reference star catalog.

4 Differential Reductions

An accurate position for a target object can be determined from the CCD (x, y) coordinates discussed in the previous section and the known equatorial positions $(\alpha_{cat}, \beta_{cat})$ of reference stars in the CCD frame. The catalog positions for the reference stars are given for a specific equinox T_{cat}, which is usually J2000. At the epoch of the CCD observation T, the coordinates for each reference star are given by

$$\alpha = \alpha_{cat} + \mu_\alpha (T - T_{cat}) \tag{7}$$

$$\delta = \delta_{cat} + \mu_\delta (T - T_{cat}) \quad , \tag{8}$$

where μ_α and μ_δ are its proper motions in right ascension and declination which are usually given in the catalog. All of the reductions must be made at the same epoch; otherwise, the proper motions will introduce errors into the reductions. Since the GSC catalog does not include proper motions, they cannot be applied; and therefore, the overall accuracy of the catalog degrades with time. The Tycho catalog will include proper motions.

After the equatorial coordinates of the reference stars are determined at the epoch of observation, they are used to compute the corresponding standard coordinates (X, Y) in the plane of the CCD, where X is directed along increasing right ascension and Y toward the nearest celestial pole. The standard coordinates are in radians. Sometimes the symbols

(ξ, η) are used for standard coordinates in the literature. As discussed in König (1962), van de Kamp (1967), and Green (1985), the standard coordinates can be computed with the following relations

$$X = \frac{\cos\delta \ \sin\Delta\alpha}{\sin\delta \ \sin\delta_o + \cos\delta \ \cos\delta_o \ \cos\Delta\alpha} \tag{9}$$

$$Y = \frac{\sin\delta \ \cos\delta_o - \cos\delta \ \sin\delta_o \ \cos\Delta\alpha}{\sin\delta \ \sin\delta_o + \cos\delta \ \cos\delta_o \ \cos\Delta\alpha} \tag{10}$$

where

$$\Delta\alpha = \alpha - \alpha_o \ . \tag{11}$$

The position (α_o, δ_o) is the point where the projected sky image is tangent to the plane of the CCD and is referred to as the tangent point. This paper will only discuss the case when the field of view defined by the size of the CCD is $\leq$ 2 degrees. With such a small field, the tangent point can be assumed to be at the centre of the field. As discussed by König (1962), larger fields of view require corrections for atmospheric refraction, annual aberration, nutation, and precession before standard coordinates can be computed accurately. These corrections can be quite complicated and will not be discussed further in this paper.

After the standard coordinates are determined for the reference stars, they can be related to the CCD coordinates (x, y) with the following linear relations

$$X - x = c_1 + c_2 x + c_3 y \tag{12}$$

$$Y - y = c'_1 + c'_2 x + c'_3 y \quad , \tag{13}$$

after the CCD coordinates in pixels are converted to radians with

$$x\ (\text{radians}) = 4.848 \times 10^{-6}\, S_x x\ (\text{pixels}) \tag{14}$$

$$y\ (\text{radians}) = 4.848 \times 10^{-6}\, S_y y\ (\text{pixels}) \quad , \tag{15}$$

and S_x and S_y are the scales of the telescope (in arcsec pixel^{-1}) in right ascension and declination. These scales do not have to be known with great precision. The scales can be determined by exposing two stars with significantly differing right ascensions and declinations on a CCD frame. Their spacing in pixels can be determined in each coordinate direction, and since the corresponding spacings in arcseconds can be determined from the known equatorial coordinates, the scales can be easily computed.

Separate solutions using Equations (12) and (13) are made with least-squares in order to determine the transformation coefficients c_1, c_2, c_3, c'_1, c'_2, and c'_3. Sometimes one or more reference stars (often blended star images or stars with poor catalog positions) will distort one of these solutions. If there are such stars, they should be removed from consideration and the calculations repeated. These stars can be identified when the residuals from the least-squares solutions are plotted against the x- and y- coordinates of

reference stars. If large residuals are present in any of the plots (herein defined as exceeding three times the standard error of the least-squares fit or, visually, as obviously very large residuals), then the reductions should be repeated with the corresponding reference stars eliminated. Ideally, the final residual plots should show only a random scatter. For example, Figure 2 shows final residuals for the asteroid 243 Ida as determined with the USNO (Flagstaff Station) 20-cm CCD transit telescope (Stone 1993). The plots show no obvious systematic trends (such as slopes or curvatures) or stars with extremely large residuals. Sometimes, poorly aligned telescopes show curvatures in the residual plots. It is also worthwhile to plot the residuals against the magnitudes of the reference stars. A linear trend could indicate poor charge transfer in the CCD or a large guiding error.

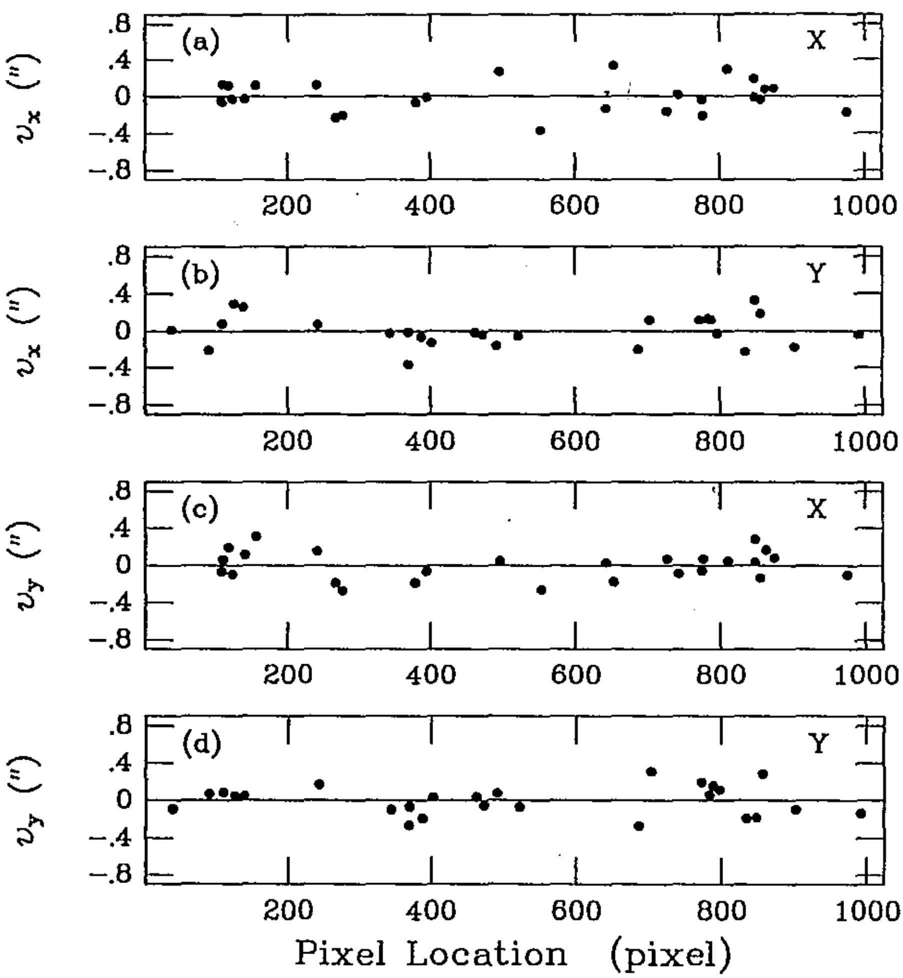

Figure 2. The distribution of least-squares residuals along the x- and y- coordinate directions is shown for the asteroid 243 Ida observed with the USNO 20-cm CCD transit telescope. Figures (a) and (b) show the distributions of residuals in right ascension, and Figures (c) and (d) show the corresponding distributions in declination. No systematic trends of unusually large residuals are apparent in any of the plots.

After the transformation coefficients are well determined, the standard coordinates of the target object can be calculated with Equations (12) and (13) and then converted to equatorial coordinates with the following relations given by König (1962)

$$\tan \Delta\alpha = \frac{X}{\cos\delta_0 - Y\sin\delta_0} \tag{16}$$

$$\alpha = \alpha_0 + \Delta\alpha \tag{17}$$

$$\tan\delta = \frac{\sin\delta_0 + Y\cos\delta_0}{\cos\delta_0 - Y\sin\delta_0}\cos\Delta\alpha \quad . \tag{18}$$

Hence, the position of the target object can be determined at the epoch of the observation. The expected positional accuracies are ±0.6 arcsec in both coordinates when reference star positions are taken from the GSC catalog, or ±0.1 arcsec referenced to the Tycho catalog. As discussed earlier, this accuracy will greatly improve with the release of new star catalogs. For solar system objects, it is also important to know the time of each CCD observation to ±1 second of time or better. This time is just the mean of the times when the shutter was opened and closed. The normally chosen time for astronomical observations is Universal Time.

Magnitudes can be often determined from the same CCD frames used for astrometry. Many of the star catalogs include magnitudes that can be used differentially to determine the magnitudes of target objects. Alternatively, ancillary observations of photometric standards can be used to determine even more accurate magnitudes using such methods as are discussed, for example, in Hardie (1962), Henden and Kaitchuck (1982), and Kaitchuck *et al* (1994).

5 Summary

Amateurs can make now significant contributions to astrometry with CCD observations. This is particularly true in the area of solar system astrometry, where amateurs are now making CCD observations. These observations are being made differentially to the GSC catalog with accuracies of ±0.6 arcsec in each coordinate for a single observation. In a few years, the Tycho catalog of star positions will become available, and afterwards, accuracies of ±0.1 arcsec should become possible. This paper has described how these reductions can be made, and what requirements are needed for good astrometry. Tables I and II show which combinations of telescope focal length and CCD pixel size are needed for the best results. Telescopes not meeting these criteria will, to varying degrees, be affected by centroiding errors and errors resulting from a lack of reference stars. Although only three reference stars are needed in astrometric reductions, in principle, this paper recommends a minimum of 10 reference stars for the best astrometry. Moreover, since atmospheric refraction is more pronounced at shorter wavelengths, telescope passbands should be chosen in the visual or at longer wavelengths. Meyer and Raab (1995) give a discussion of CCD astrometry for amateurs as well. This paper gives more detail and particularly of the requirements needed for good astrometry.

6 References

Chui, L.-T.G., 1977, *Astron. J.*, **82**, 842.

Graham, J.A., 1982, *Pub. Astron. Soc. Pacific*, **94**, 244.

Green, R.M., 1985, *Spherical Astronomy*, Cambridge, Cambridge University, 310.

Gunn, J.E. and Knapp, G.R., 1993, in *Sky Surveys: Protostars to Protogalaxies*, ASP Conference Series, **43**, ed. B.T. Soifer, 267.

Hardie, R.H., 1962, in *Astronomical Techniques, Stars and Stellar Systems*, **2**, ed. W.A. Hiltner, Chicago, University of Chicago, 178.

Henden, A.A. and Kaitchuck, R.H., 1982, *Astronomical Photometry*, New York, Van Nostrand Reinhold.

Høg, E. *et al* 1992, *Astron. Astrophys.*, **258**, 177.

Holmes, A., 1995, *CCD Astronomy*, **2**, 14.

Janesick, J. and Blouke, M., 1987, *Sky and Telescope*, **74**, 238.

Kaitchuck, R.H., Henden, A.A. and Truax, R., 1994, *CCD Astronomy*, **1**, 20.

König, A., 1962, in *Astronomical Techniques, Stars and Stellar Systems II*, ed. W.A. Hiltner, Chicago, University of Chicago, 461.

Kristian, J. and Blouke, M., 1982, *Scientific American*, **247**, 66.

Lindegren, L., van Leeuwen, F., Petersen, C., Perryman, M.A.C. and Söderhjelm, S., 1992, *Astron. Astrophys.*, **258**, 134.

Lopez, G.A. and Yagudin, L.I., 1995, in *Dynamics, Ephemerides and Astrometry in the Solar System*, IAU Symposium No. 172 (in press).

Mackay, C.D., 1986, *Ann. Rev. Astron. Astrophys.*, **24**, 255.

Marsden, B., 1988, *Sky and Telescope*, **76**, 462.

Meyer, E. and Raab, H., 1995, *CCD Astronomy*, **2**, 20.

Monet, D., Canzian, B. and Henden, A., 1994b, *Bull. Amer. Astron. Soc.*, **26**, 1314.

Morgan, D.H., Tritton, S.B., Savage, A., Hartley, M. and Cannon, R.D., 1992, in *Digitized Optical Sky Surveys*, ed. H.T. MacGillivray and E.B. Thomson, Dordrecht, Kluwer, 11.

Russell, J.R., Lasker, B.M., McLean, B.J., Sturch, C.R. and Jenkner, H., 1990, *Astron. J.*, **99**, 2059.

Stone, R.C., 1989, *Astron. J.*, **97**, 1227.

Stone, R.C., 1993, in *Developments in Astrometry and their Impact on Astrophysics and Geodynamics*, IAU Symposium No. 156, ed. I.I. Mueller and B. Kolaczek, Dordrecht, Kluwer, 65.

van den Kamp, P., 1967, *Principles of Astrometry*, San Francisco, W.H. Freeman, 73.

van Altena, W.F. and Auer, L.H., 1975, in *Image Processing Techniques in Astronomy*, ed. C. de Jager and H. Niewenhuijzen, Dordrecht, Reidel, 411.